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Flow separation at the upstream edge of a square-edged broad-crested weir

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A simple model is suggested to explain the flow mechanism at the upstream edge of a square-edged broad-crested weir. The separation bubble that may be seen to occur at this point is treated as an area of constant static head, while the main flow outside the bubble is deemed to be irrotational and divided from the bubble by a free streamline. If this model is accepted, the flow pattern will be such that energy requirements will be met along the free surface and the streamline bounding the bubble while, within these boundaries, the Laplace equation will be satisfied at every point. Accordingly, a solution satisfying these conditions is established by the use of a relaxation technique.

In practice, it is likely that the cavity flow within the bubble will be bounded not by a single streamline but by a turbulent mixing zone and that there will be some increase in pressure near the point of re-attachment. Nevertheless, the surface profile and flow pattern observed in experiments show fair agreement with those predicted using the simple model. Whilst acknowledging, then, that the bubble and the zone bounding it are in fact of a more complex character, we may say that this simplified treatment affords a sound model of the main flow and so permits a better understanding of the action of the square-edged broad-crested weir.

1. Introduction

When water flowing in an open channel passes over a positive step with a square edge the main flow does not follow the sharp corner, but takes a path of finite radius before re-attaching itself to the floor some distance downstream. A region of cavity flow, a separation bubble or roller, is thus formed near the edge. The pattern may readily be seen if the flow is well illuminated and contains reflecting particles; in figure 1 (plate 1) it has been made visible by the introduction of fine air bubbles. Alternatively, it may be traced by the use of fine threads anchored in the flow, as was done by Keutner (1934).

This, then, is the situation that occurs at the upstream edge of a broad-crested weir with a free overfall at its downstream end. In the simplest one-dimensional treatment of the problem, however, such complicating factors as separation and flow curvature are ignored and the velocity distribution is treated as uniform all along the crest. Bélanger (1849), using these simplifications, equated the discharge to the maximum possible for the total upstream head H which, for an approach

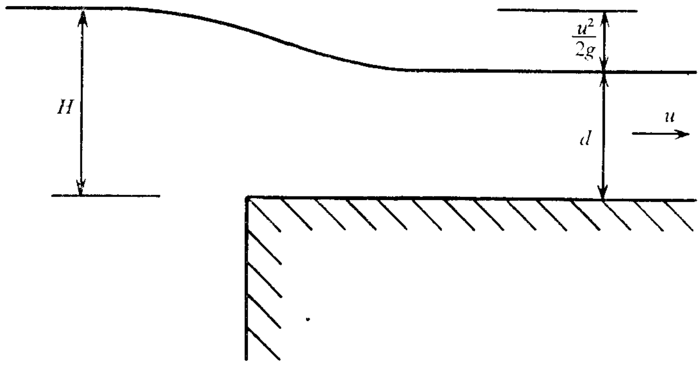

FIGURE 2. Simplified flow diagram.

channel of infinite depth, is simply equal to the height of the upstream surface above crest level (figure 2). From Bernoulli's equation

$$H = d + u^2/2g, \tag{1}$$

where d is the depth on the crest and u is the velocity. The crest depth for maximum discharge, the critical depth, may readily be shown to be $\frac{2}{3}H$. The corresponding discharge per unit width would be $g^{\frac{1}{2}}(\frac{2}{3})^{\frac{3}{2}}H^{\frac{3}{2}}$ but in practice the measured discharge is found to differ appreciably from this value and is commonly expressed as $Cg^{\frac{1}{2}}(\frac{2}{3})^{\frac{3}{2}}H^{\frac{3}{2}}$, where C is a coefficient of discharge.

The principle of momentum might be applied provided that some assumptions were made as to pressures on the upstream face of the weir. Doeringsfeld & Barker (1941), in their experimental investigation of the square-edged weir, found that, with little error, the distribution of these pressures might be taken as equal to the hydrostatic pressure distribution upstream, substantially as though the velocity heads at the face were negligible. If this result were adopted and the horizontal force were then equated to the change in momentum per unit time for the case of infinite approach depth (forces below crest level being balanced) then

$$\tfrac{1}{2}\rho g H^2 - \tfrac{1}{2}\rho g d^2 = \rho u^2 d. \tag{2}$$

Eliminating u between (1) and (2) would then yield the result

$$3d^2 - 4dH + H^2 = 0, \tag{3}$$

$$d = H \quad \text{or} \quad \tfrac{1}{3}H. \tag{4}$$

If the first root were discarded as corresponding to zero discharge, this approach would yield a value for the crest depth of $\frac{1}{3}H$, a very different result from the previous figure of $\frac{2}{3}H$. In fact, Doeringsfeld & Barker found measured values in the region of $\frac{1}{2}H$, half-way between the two figures.

A one-dimensional treatment of the problem is clearly inadequate. For a fuller insight the two-dimensional flow pattern involving separation must be studied. With separation the situation is considerably modified; Rouse (1936) likened the roller to a small spillway built upon the main weir so that beyond the roller the velocity will be rather greater than critical. The form of the roller is not fixed independently but is itself one feature of the flow pattern. The aim of

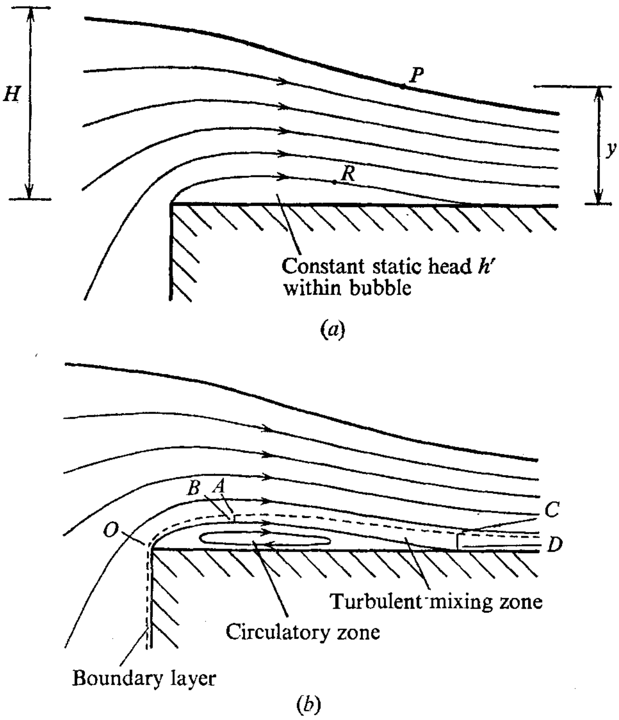

FIGURE 3. Models of flow mechanism (a) with constant static head in bubble and (b) with circulatory motion in bubble bounded by turbulent mixing zone.

the work now described has been to produce an analytical basis to account for that pattern. (The treatment will not be applicable where a free overfall does not exist, nor in those special cases where the weir is either so short that no stretch of parallel flow can be accomodated or so long that, owing to the effects of friction, subcritical flow, controlled from the downstream end and characterized by the presence of waves, drowns and modifies the pattern at the upstream end.)

2. Flow pattern

If the viscosity of the water is neglected, the main flow outside the separation bubble may be deemed irrotational and thus amenable to treatment by potential-flow theory. Analysis can only be attempted, however, if it is possible to define boundary conditions for this flow. In fact a mixing process must take place along the bubble. Nevertheless, the proposal is now put forward that for the purpose of analysing the main flow the bubble should be treated as though it contained water with negligible velocity under a constant static head; i.e., that the boundary of the bubble should be treated as a free streamline (figure 3(a)).

The real situation must clearly be more complex; there must be some interchange of mass and momentum between the bubble and the flow outside. Rouse (1960), when investigating the rather similar flow situation around the end of a blunt cyclindrical shaft, declared that the line of separation would not be a free streamline but that it would be a line along which the energy steadily decreased. Highly turbulent fluid would be re-entrained from the bubble, the downstream end of which would fluctuate so as to throw off parts of itself into the main stream, the process being accompanied by a dissipation of energy.

The bubble is, then, likely to be circulatory in character rather than static, although the flow within it will be complex (figure 1) and will vary with time so that turbulence will be high although mean velocities and the corresponding velocity heads may not be great. Squire (1956) suggested that a region of re-circulation might usefully be conceived as having a core with a boundary layer around it, motion within the core being maintained by shear stresses caused by the outer flow acting on this layer, the core itself being defined as a region where viscous effects are virtually absent.

The situation seems likely to have some affinity with leading-edge separation on wings. A more realistic model of the separation zone, therefore, would probably be as shown in figure 3(*b*), which follows the simplified model for leading-edge separation proposed by Crabtree (1957), amplifying Squire's suggestion. The boundary layer will already be of finite thickness at the edge of the weir O, but the main turbulent mixing will probably occur as the layer increases from a thickness AB at the highest point of the zero streamline or mean dividing line to a thickness CD at the point of re-attachment. Although the static head may rise from AB to CD, it should change little between O and B, for Crabtree, finding in his study of leading-edge separation a constant pressure over the forward part of the bubble, suggested that it should be possible in that region to retain some of the assumptions of boundary-layer theory, including that of no variation of the static pressure normal to the solid surface.

Again, for the related problem of separation and re-attachment downstream of a single-step roughness element, Mueller, Korst & Chow (1964) took a simplified theoretical model assuming a quiescent wake behind the step. The pressure distribution along the wall was found to exhibit three distinct zones, free-jet mixing at constant pressure, re-attachment against rising pressure and re-development at essentially constant pressure. A similar situation might be expected in the present case, but would be modified by the fact that with the free surface there is a pressure gradient in the main flow falling in the downstream direction.

Up to the widest part of the bubble, then, the simplest hypothesis proposed differs but little from the more highly developed models. It is, moreover, the condition in this region that should determine the rate of discharge. Further downstream where velocities have become supercritical, local modifications to the pressure distribution should have only secondary effect. (This point is considered further in the next section.)

The simplifying assumption, whereby the bubble is taken as a static zone bounded by a free streamline, is therefore advocated as affording a simple basis for calculation not greatly at variance in its effect upon the outer flow with the concepts of previous writers. It is acknowledged that the flow might properly be divided into three regions, namely the external flow virtually irrotational in character, the circulatory zone within the bubble and the turbulent mixing zone in between, and that the solutions for each must be matched. Nevertheless, an approximate solution for any one region, in this case the outer flow, may be obtained if the effect upon it of the other two is adequately presented, although the behaviour of those two zones has not been fully resolved in detail.

With the simplified flow model proposed, therefore, an analytical treatment has been developed and its results compared with experiment.

3. Computation

For a two-dimensional pattern of irrotational flow within fixed boundaries the Laplace equation must be satisfied at every point:

$$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 = 0. \tag{5}$$

The fixed boundaries will themselves be streamlines so ψ has a constant value along each boundary and within the boundaries there is a unique solution for the flow pattern which may be determined exactly for some simple cases. The particular problem that arises with free surfaces is that the boundaries are not initially given. They must, however, be such as to satisfy certain conditions. As is the case for fixed solid boundaries, no flow crosses such a surface, and the surface, therefore, may be called a free streamline. Further, if there is no energy degradation Bernoulli's equation must be satisfied at all points along the line. These added conditions, then, ensure that even where a free surface exists and motion is influenced by gravity there is still a unique solution. Thus, if at point P (figure 3 (a)) the height of the surface above crest level is y and the velocity is q_P with total head on the weir H, then, from Bernoulli's equation,

$$q_P = (2g(H-y))^{\frac{1}{2}}. \tag{6}$$

When separation occurs, as it does at the entry to the square-edged weir, within the separation zone, flow cannot be deemed irrotational and Laplace's equation cannot be applied there. If, however, the outline of the separation zone is taken as a free streamline, the area outside this line may still be treated as irrotational. If, further, the bubble be treated as though it contained water at rest under a constant static head of h' then at point R (figure 3 (a)) on the outline of the bubble, the velocity q_R is given by Bernoulli's equation:

$$q_R = (2g(H-h'))^{\frac{1}{2}}. \tag{7}$$

Although, with these simplifications, the necessary boundary conditions are readily specified, the solution cannot be found by direct analysis. The device of conformal transformation can, in some cases, be used to determine a flow pattern directly and its use has been extended to cases where a free surface exists, but it cannot well be applied in the present instance, where the motion is influenced by gravitational forces, and, furthermore, separation occurs. Such situations can, nevertheless, be treated by relaxation methods such as that developed by Southwell (1946) and it is a treatment of this type which is now described.

Basically, boundaries are assumed together with values of ψ at each point of a regular grid and the ψ values are successively adjusted or relaxed until the Laplace equation, expressed in finite-difference form, is satisfied at every point.

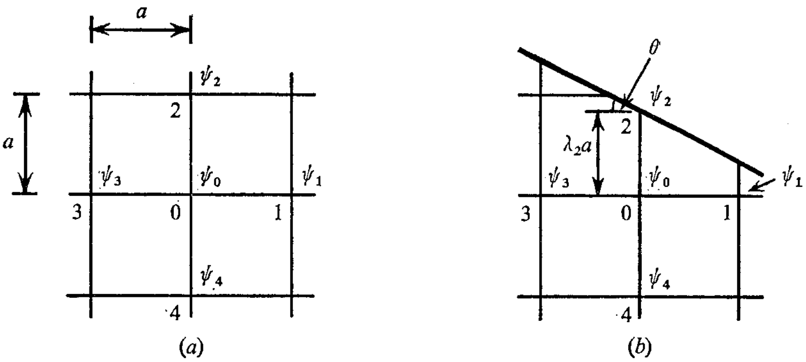

FIGURE 4. Stream function values (a) for regular star and (b) for irregular star.

The velocities at the boundaries are then calculated; if they do not everywhere satisfy Bernoulli's equation to a certain degree of accuracy the boundaries are revised and the process is repeated as many times as may be necessary.

In terms of finite differences, the Laplace equation has a simple form. If the area under consideration is covered by a square grid of lines, distance a apart, and at a point O on the grid (figure 4(a)) ψ has the value ψ_0, while at the four adjacent points 1, 2, 3 and 4 the values are ψ_1, ψ_2, ψ_3 and ψ_4, respectively, then, if the grid is sufficiently fine the following approximation may be accepted:

$$\psi_0 = \tfrac{1}{4}(\psi_1+\psi_2+\psi_3+\psi_4). \tag{8}$$

If one or more of the relevant strings of the grid is cut by a boundary so that an irregular cross or star is formed (figure 4(b)) then a modified form of the equation applies:

$$\psi_0 = \frac{\psi_1+\psi_2/\lambda_2+\psi_3+\psi_4}{3+1/\lambda_2}, \tag{9}$$

where the distance of one of the points, 2, from point O is in this case $\lambda_2 a$.

The area taken for calculation was bounded by vertical lines at distances $2H$ upstream and downstream, respectively, from the edge and by a horizontal line distance $2H$ below crest level (figure 5). The region so defined included all areas of sharply curved flow. Trial lines for the surface and for the bubble outline were adopted; it was assumed that with frictionless flow and a long crest the depth would approach a value d at some point downstream where the depth and velocity become uniform. ψ was given the value zero along the crest and 1000 along the surface, and at each point of a square grid of side $0{\cdot}2H$ intermediate values were assigned.

Then, working systematically through the grid, the values of ψ at each point on the grid were replaced by the values calculated from equation (8) or (9), whichever was appropriate, the whole cycle being repeated until no correction made within a cycle was greater than a chosen minimum value. A figure of 1·0 was taken as giving, in general, acceptable accuracy. Greater accuracy seemed desirable in the area of very sharply curved flow near the inlet edge. For an area bounded by horizontal and vertical lines $0{\cdot}2H$ from the edge, therefore, a finer square grid of side $0{\cdot}04H$ was used and the relaxation process was continued to yield an accuracy of $\pm 0{\cdot}2$.

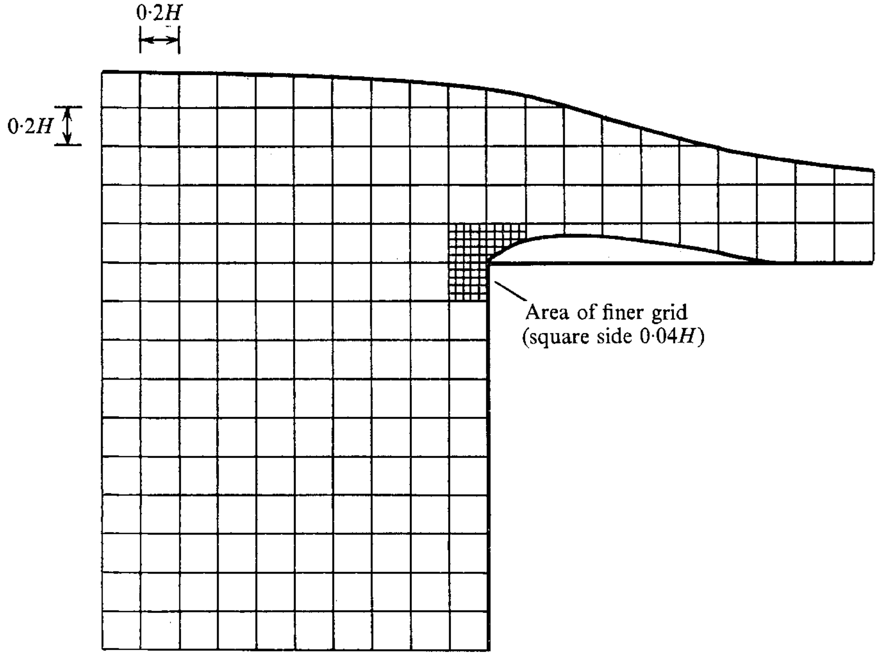

FIGURE 5. Grid for relaxation technique.

When the relaxation process was complete, the velocity at every point where a vertical grid line intersected the surface or the bubble outline was calculated, again using a finite-difference expression.

Generally

$$q = k\,\partial\psi/\partial n, \tag{10}$$

where k is a constant and n is distance measured perpendicular to direction of flow. Where the flow becomes uniform downstream, at depth d,

$$q = [2g(H-d)]^{\frac{1}{2}} \tag{11}$$

and

$$\partial\psi/\partial n = 1000/d. \tag{12}$$

Thus k may be evaluated and (10) re-written for the general case

$$q = \frac{d}{1000}[2g(H-d)]^{\frac{1}{2}}\frac{\partial\psi}{\partial n}. \tag{13}$$

Then at any point on the boundary such as 2 in figure 4(*b*)

$$q = \frac{d}{1000}[2g(H-d)]^{\frac{1}{2}}\frac{\partial\psi}{\partial y}\sec\theta. \tag{14}$$

An estimate of $\partial\psi/\partial y$ is given by $(\psi_2-\psi_0)/\lambda_2 a$. A better approximation, recommended by McNown, Hsu & Yih (1955) is

$$\frac{\partial\psi}{\partial y} = \frac{1+\lambda_2}{\lambda_2}\frac{\psi_2-\psi_0}{a} - \frac{\lambda_2}{1+\lambda_2}\frac{\psi_2-\psi_4}{a}. \tag{15}$$

If Bernoulli's equation was not satisfied throughout (to an accuracy of $\pm\,0{\cdot}01H$) the outlines of the surface and the bubble were revised and the process was repeated until a solution was attained, the aim being to establish the outlines which corresponded to a situation satisfying Laplace's equation at all internal points and Bernoulli's equation along the free streamlines.

The calculation, which is highly repetitive but made up of simple elements, is best undertaken on the digital computer and this was done in the present case. The solution was found by trial, each revision of the boundaries being made by hand and the relaxation process and calculation of velocities alone then being done by the computer. It would have been desirable to incorporate the revision of the boundaries also into the computer program, but it was difficult to formulate sufficiently precise rules for achieving an improvement. A change in any part of a boundary modified the flow pattern over a considerable area; to raise the surface locally at some point, by changing the curvature, altered the velocity distribution and, further, could either increase or decrease the total head at the point according to whether the velocity there was relatively low or high. The form of the bubble was gradually evolved so as to give a constant velocity along the outline, neither the velocity nor the outline having been initially known.

The method of working, then, was to begin with a value for the uniform downstream depth d of $\frac{2}{3}$, as given by Bélanger's approach, and to attempt to derive a solution consistent with the physical equations. (Sketches of outlines and flow nets were useful as a preliminary to detailed work with the computer.) It did not, in fact, prove possible to obtain a solution with this value of d and progressively lower values were therefore tried, implying correspondingly lower rates of discharge. Ultimately a value was reached for which, by a series of tentative modifications to the boundary, with each choice influenced by the effect of previous ones, a solution was achieved satisfying the assumed conditions. This then was accepted and subjected to comparison with experiment.

The accepted solution for the main grid points as derived and printed by a ICL 1905F computer is shown in figure 6. At each internal point on the main grid the value of ψ is shown. At the intersection of each vertical grid line with the surface the surface level is shown with, beneath it, the level needed to satisfy Bernoulli's equation, both expressed as a fraction of H above crest level and agreeing with each other to an accuracy of $\pm 0{\cdot}01H$. Similarly, at the intersection of each vertical grid line with the bubble outline there is shown the level of the outline and the static head within the roller satisfying Bernoulli's equation. This static head was found to have the value $0{\cdot}60$–$0{\cdot}61H$, while the uniform depth approached downstream was $0{\cdot}45H$, comparable with the measured value of $\frac{1}{2}H$ of Doeringsfeld & Barker (1941) already quoted.

A solution having been obtained assuming constant pressure within the bubble, thought was given to the effect of modifying conditions at the downstream end to accord with some measure of pressure recovery towards re-attachment. The outline of the upstream part of the bubble was not changed but the downstream part was tapered more sharply to zero thickness so that the total length of the bubble was $1{\cdot}4H$ instead of $1{\cdot}55H$. With this new boundary for the bubble fixed, the free surface was adjusted as necessary, to satisfy Bernoulli's equation throughout. This modification yielded a solution in which the pressure in the bubble, while still constant in the upstream half at $0{\cdot}60$–$0{\cdot}61H$, rose towards $0{\cdot}67H$ at the

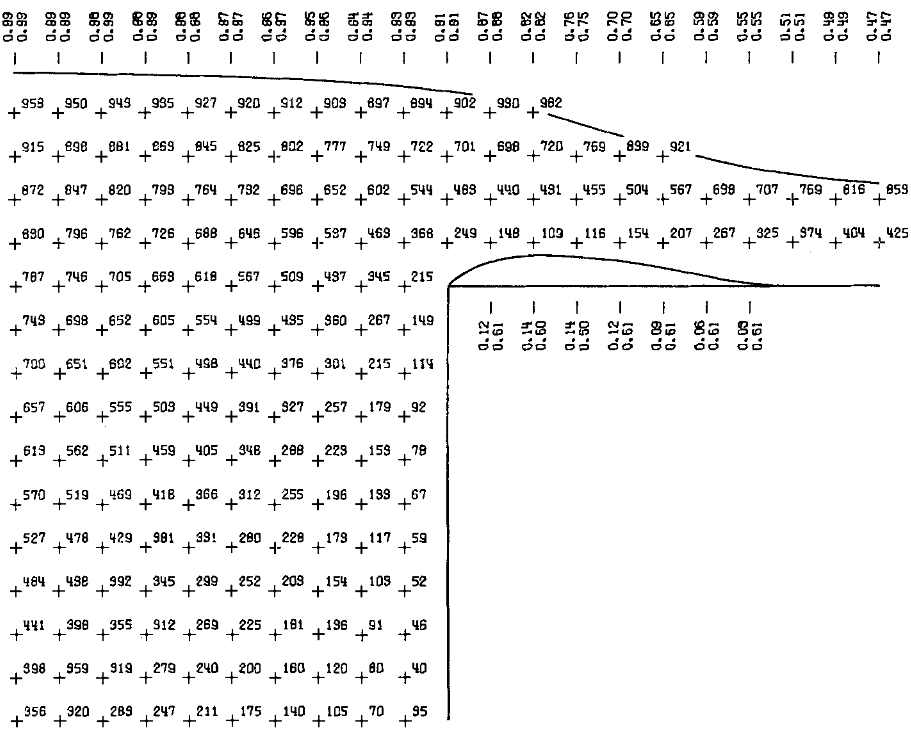

FIGURE 6. Computer print-out of accepted solution (see text for details). Main grid only shown, boundaries added by hand.

extreme tail. While some changes had been made in the free surface levels over the end of the bubble none were needed further upstream nor was the uniform downstream depth changed. It seems, therefore, that any pressure rise at re-attachment, ignored in the simplified treatment now proposed, should have only a local effect. The pressure rise should, in any case, be less than in a semi-infinite fluid for with the free surface the pressure in the main stream, instead of remaining at H, falls steadily to a value less than $\frac{1}{2}H$.

4. Experiments

In order to check the validity of the analysis, the surface profile, flow pattern and pressure distribution were observed with square-edged broad crested weirs set in a horizontal laboratory flume 610 mm wide. Three weirs were made of Perspex and were all of similar construction, a square-cut sheet for the crest being cemented on to vertical end-walls, with all joints finished flush; the weirs, stiffened by internal ribs, were all 152 mm high and, respectively, 152 mm, 381 mm and 762 mm long. When in use, a weir was sealed in position in the flume with its crest horizontal. The flume was fitted with side rails along which transverse instrument carriers could travel. The water supply was self-contained, the water being returned by a pump from the downstream end to a header tank fitted with baffles and vanes to yield as uniform a flow as possible upstream of the weir; the rate of flow was controlled by regulating valves. Water levels in the flume could

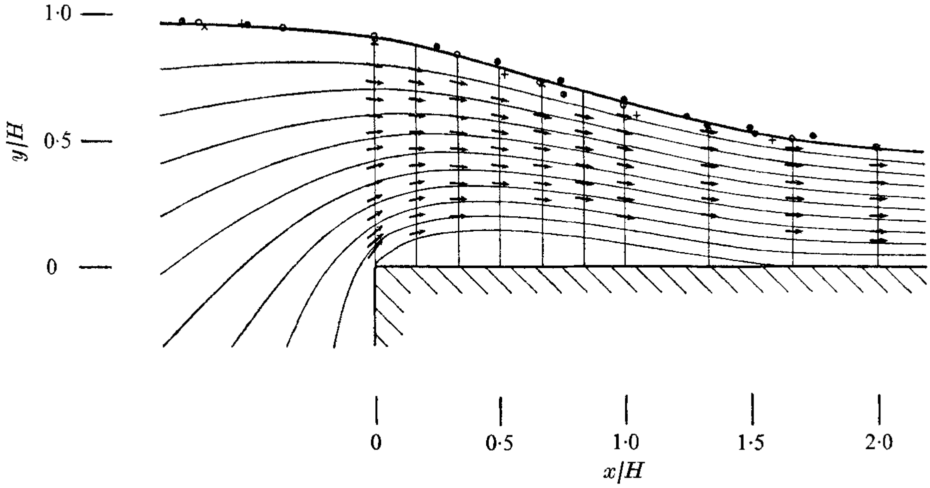

FIGURE 7. Computed profile and streamlines with observations superimposed. →, observed velocity directions.

Observed surface levels	Head H(mm)	Crest length L (mm)	L/H
○	76	152	2
●	51	152	3
×	38	381	10
+	48	762	16
◑	33	762	23

be measured by means of a point gauge attached to a vertical traveller on the transverse instrument carrier, scales with verniers facilitating horizontal and vertical measurements.

With various heads, a series of surface profiles in the neighbourhood of the upstream edge were recorded for each of the three weirs, using the point gauge. The range of working chosen was such that, in every case, the head was neither so great as to give a profile falling with pronounced curvature throughout nor so small as to cause the supercritical conditions near the upper end to be drowned owing to the effects of friction further down the crest.

With a head of 76 mm on the weir of length 381 mm the flow pattern was established by measuring the mean-velocity direction at a number of points by means of a miniature Pitot cylinder. This cylinder was formed from a stainless-steel tube, 1·5 mm external diameter, plugged at its mid-point with a rubber insert and having two holes drilled in the tube at points 12 mm on either side of the plug, the radii through the centres of the holes making an angle of approximately 30° with each other. The tube was supported in a horizontal position at right angles to the flow in a suitable frame carried, as was the point gauge, on the vertical traveller of the instrument carrier. Each end of the tube was connected to an open-ended standpipe, the level in which could be observed with a travelling microscope, and one end carried a pointer registering against a protractor. Thus, the zero reading having been established, it was subsequently possible to measure the direction of the velocity vector at any point to an accuracy of $\pm 1°$, and a series of vertical traverses were made at points along the crest near

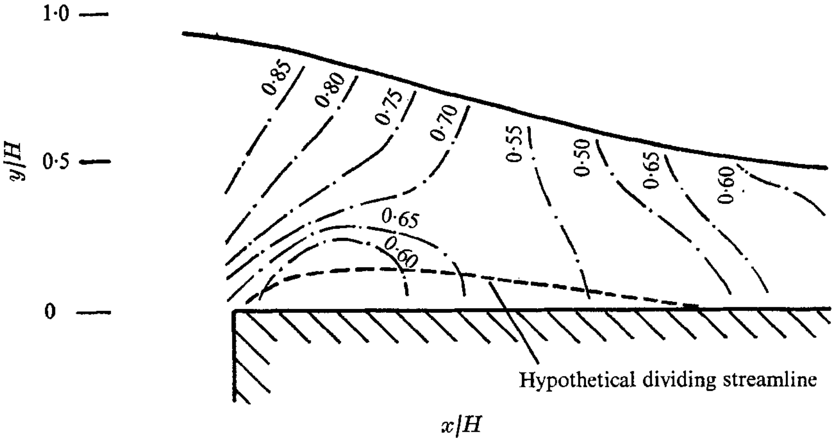

FIGURE 8. Distribution of observed pressures expressed as a fraction of H (with head 76 mm on weir of length 381 mm).

the inlet end. Along the same vertical sections the static pressures were measured using the static tappings of a Pitot-static tube of external diameter 2·3 mm.

In figure 7 the surface profile, bubble outline and streamlines as derived from the analysis are shown; the computer was used to calculate and plot the streamlines by interpolating from the grid values of ψ shown in figure 6. The experimental results for surface level and velocity direction are shown superimposed. (The depth of approach in the experiments was finite but, nevertheless, with the greatest head used (76 mm) the velocity head was so small that the total head exceeded the measured head by little more than 1 %. The analysis was in fact repeated for the case where the height of the weir was not infinite but equal to twice the head on the weir. The surface profile and the flow pattern in the area over the weir were found not to differ materially, to the degree of accuracy to which the computation was performed, from the case with infinite approach depths.)

The surface levels for the varying experimental conditions lay quite close to the line predicted and, over much of the area of inviscid flow outside the bubble, the measured velocity directions agreed quite well with the computed streamlines to the degree of accuracy involved in the calculation and the experiment. Within the predicted outline of the bubble and within a certain band outside it, the measured values did not merely lack close agreement with the analysis but were generally erratic and unsteady and none, therefore, have been shown in this region. The effect seems to be consistent with the existence of a high degree of turbulence in the region of cavity flow and in the mixing zone bounding it. Tani (1958), investigating the cavity flow caused by separation over a step, acknowledged the difficulty of measuring the mean velocities which, in such a situation, are small while, at the same time, the fluctuations of velocity are large.

The measured distribution of static head is indicated in figure 8. The analysis, with its basic simplifying assumptions, implied a free streamline bounding the separation zone with a constant static head of 0·60–0·61H. Over the upstream half of this line the measured static head was found to have a value of 0·56–

$0{\cdot}58H$, almost constant although slightly lower than the computed value. Downstream from the highest part of the bubble, the static head along the path of the hypothetical dividing streamline first rose towards a value of $0{\cdot}69H$ before falling back, under the falling surface profile, to $0{\cdot}60H$. A certain rise in pressure at re-attachment is therefore indicated.

5. Conclusion

The present aim has been to gain an improved insight into the nature of the flow over the upstream corner of a square-edged broad-crested weir. It is advocated that, as a useful approximation, the separation bubble that occurs should be treated as an area of constant static head divided by a free streamline from the main flow, which is then to be considered irrotational. The main flow pattern which is implied by this assumption and which may, with this simplification, be determined by analysis, agrees well with observation outside the immediate vicinity of the hypothetical dividing streamline. It is accepted that this single line will in practice be replaced by a turbulent mixing zone of finite thickness. This thickness will increase over the downstream part of the bubble where some recovery of static head may be expected and is, in fact, found to occur, in conjunction with evidence of pronounced turbulence. Over the upstream part of the bubble the effect is less pronounced and the observed flow pattern near to the dividing streamline accords better with the analysis. It is in this region, where the bubble reaches its greatest height, that the control will be established, determining the rate of discharge for a given head. Further downstream, where velocities will have become supercritical, friction, neglected in the simplified treatment, will become significant but this will not affect the pattern near the edge nor the rate of discharge.

Acceptance of the concept of a control established over the separation bubble, itself of fixed outline for a given head, implies a constant discharge for that head or, in non-dimensional terms, a fixed coefficient of discharge, unless conditions completely alter the situation at the upstream edge. If the weir is too short then, clearly, the bubble cannot be accommodated in unmodified form. If the weir is too long, subcritical conditions from the downstream end may drown the flow at entry so that critical conditions cannot exist there. If neither of these extremes prevail, however, it may be expected that the coefficient of discharge will be constant and, indeed, the results of various investigators (such as Horton (1907), who summarized earlier work in addition to providing new results) have indicated that this is the case. The results have shown some variation both between workers and within individual investigators but, in general, all demonstrate a range with almost constant coefficient ; a mean figure might perhaps be taken as 0·85 with a scatter of $\pm$ 0·03 about this mean. This may be compared with the figure obtained from the analysis where, with the uniform depth d downstream shown to be $0{\cdot}45H$, the flow per unit width would be given by

$$d\,[2g(H-d)]^{\frac{1}{2}} = 0{\cdot}87g^{\frac{1}{2}}\,(\tfrac{2}{3})^{\frac{3}{2}}H^{\frac{3}{2}},$$

implying a coefficient of 0·87, a value within the range of previous experimental findings.

The approximate treatment advocated may, then, yield useful results, especially if the particular concern is with the irrotational main flow. A sounder basis for the assumptions as to boundary conditions could probably be achieved, however, if a fuller knowledge were obtained of the distribution of mean velocity, turbulence and energy. The relaxation technique has the merit that it may readily be adapted for use with a variety of boundary conditions which may be specified in various ways, not only by shape but by velocity or pressure distribution. Such conditions may be based upon figures obtained by experiment but the analysis would be more comprehensive if the boundary conditions were implicit in the model developed for the overall flow pattern. A similar technique for calculation may be applied to other situations where separation and re-attachment occur. In turn, investigations of other situations of this type, such as that of Rouse (1960), dealing with flow around a blunt-ended cylinder, or Butté & Pichon (1970), dealing with flow over a backward facing step in an open channel, may help to yield a better understanding of the flow over the square edge of the broad-crested weir.

Owing to the complex character of the flow pattern, a single investigation can scarcely produce a complete analytical solution which covers the whole pattern and is yet free from approximation. Improved understanding is most likely to result from a continual combination of analysis and experiment, each serving to guide the other. The present paper is intended to suggest a provisional simplification which, while serving to stress some essential characteristics of the main flow, permits the development of an analytical treatment yielding results corresponding well with observation .

The work described was carried out at the University of Surrey using the ICL 1905F machine of the Computing Unit and experimental facilities of the Departments of Civil and Mechanical Engineering. I wish to thank Mr J. W. Wielogorski who, in supervizing the work, offered help and encouragement throughout.

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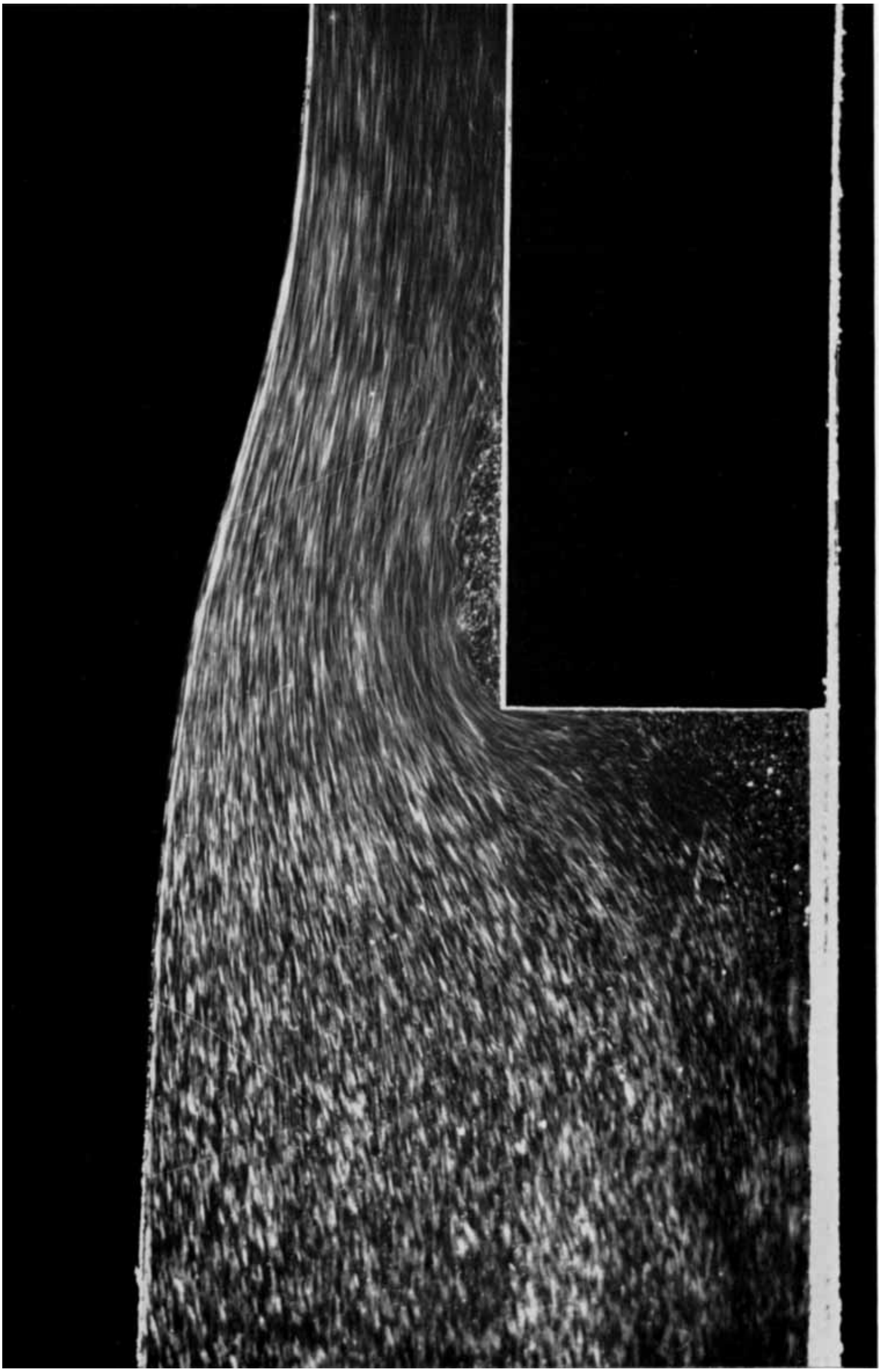

FIGURE 1. Flow separation made visible by entrained air bubbles.